FUSION RESEARCH CENTER

TAERF-2 FRCR #335

Physics Scaling of Reactor Plasmas

R J. Bickerton
Fusion Research Center
The University of Texas at Austin
Austin, Texas 78712

April 1989

THE UNIVERSITY OF TEXAS

Austin, Texas

Physics Scaling of Reactor Plasmas

R. J. BICKERTON
Fusion Research Center
The University of Texas at Austin
Austin, Texas 78712

Reactor Plasma

The plasma parameters of a reactor plasma for a steady state D-T system are narrowly defined by the technical constraints of power loading on the first wall (3-5 $\mathrm{MW/m^2}$) and the combined thickness of blanket and shield ($\sim$ 1-2m). The key plasma parameters are then $\bar{T} \sim 10$ keV, $\bar{n} \sim 1\text{–}2.10^{20}\mathrm{m}^{-3}$ and a $\sim$ 2-3m. ($\bar{T}$ is the mean plasma temperature, $\bar{n}$ the mean density, and a the minor radius.) These numbers are independent of the confinement system provided only that it is steady state. For the tokamak case the combination of engineering feasibility and the Troyon P-limit lead to aspect ratio $A \sim 4$, toroidal field $B \sim 6\,\mathrm{T}$, cylindrical $q \sim 2$, and plasma elongation $\varepsilon \sim 2$. We thus arrive at fairly well prescribed plasma conditions. Now ideally before building such an expensive device we would like to test the physics in a scaled manner, by analogy with wind tunnel or ship tank tests in other fields.

Physics of Fully-Ionized Plasma

The scaling of the physics of fully-ionized plasma was considered by Bickerton and London (1958), Kadomtsev (1975), and by Connor and Taylor (1977). The last authors gave the most complete analysis in terms of particular plasma models. Since we do not know which plasma models are operative and since in any case the appropriate model may depend on radial position we adopt the general approach of Kadomtsev in which plasma behavior is assumed to be a function of four dimensionless variables together with geometric variables such as q, A, and ε. The four dimensionless variables are

$$\beta = nT/B^2 = \text{plasma beta}$$

$$G = T/\left(B^2L^2\right) = (\text{ratio gyroradius to size})^2$$

$$C = n/\left(T^{3/2}B\right) = \text{ratio collision frequency to gyrofrequency}$$

$$D = T/\left(nL^2\right) = (\text{ratio Debye distance to size})^2.$$

With four dimensionless variables and four parameters (n, L, B, and T) characterizing the plasma it is clear that no scaling is possible and the only physics test of a reactor plasma is at full size. However if we can discard one variable as unimportant then some scaling is feasible. We discard D on the grounds that it is very small and does not appear in any theories of plasma transport due to turbulence. Then there is a tokamak "family" in which if we vary B *we* find

$$L \; cc \; B^{-4/5}, \qquad n \propto B^{8/5}, \qquad \text{and} \qquad T \propto B^{2/5}.$$

In this family of tokamaks since all time scales have the same relationship to each other we can use the simplest normalization for the confinement time τ_ε, namely $(\tau_\varepsilon\omega_c) = \text{constant}$, giving

$$\tau_\varepsilon \propto B^{-1}.$$

Fusion Reactions

In the reactor plasma we expect to be operating at the ignition point where the o-power just balances the losses from the plasma. Thermonuclear reactions are not plasma physical phenomena and therefore are not obviously conserved among the family of tokamaks discussed above. The relevant parameter is

$$F = \frac{\text{o-particle power}}{\text{plasma losses}} .$$

For the range of temperatures of interest, namely $7 < T < 23$ keV,

$$F \propto nT\tau_\varepsilon f(1 - f),$$

where $f = n_D/(n_D + n_T)$ is the mix of deuterium tritium isotopes. So for tokamaks of the same family we find

$$F \propto B(f(1 - f))$$

So if we wish all members of the family to be similar in all respects, including fusion power, then we must adjust the isotopic mix so that

$$f(1 - f) \propto B^{-1}.$$

React or Relatives

We can now tabulate the reactor plasma parameters together with its similar relative at high field. The reactor parameters are assumed to span a density range of three, corresponding to ignition at low density and then transition to a higher density operating point. Seeking the most distant credible relative we take 20 T for the high magnetic field case. The results are shown in Table 1. Evidently the high field relative of a reactor must operate at higher density, higher temperature, higher current and will ignite with a non-optimum isotope mix of 8:92 percent. The mix could be varied between predominantly deuterium to predominantly tritium in order to test the effect of mean ion mass.

	B(T)	u(m)	I(MA)	$\bar{n}\,(10^{20}\mathrm{m}^{-3})$	$\bar{T}$(keV)	$F(f = 0.5)$	$f(F = 1.0)$
Operating Reactor	6	2.5	18	1.5	10	1.0	0.5
High Field Relative	20	0.95	23	10.3	16	3.3	0.08
Igniting Reactor	6	2.5	18	0.5	10	1.0	0.5
High Field Relative	20	0.95	23	3.4	16	3.3	0.08

Table 1

a-particle physics

Despite these measures to ensure similarity there is still some residual variation in the o-particle parameters which has its origin in the' constancy of the o-particle source energy.

Thus from energy balance

$$\frac{n_\alpha \varepsilon_\alpha}{\tau_s} = \frac{nT}{\tau_\varepsilon},$$

where τ_s is the o-particle slowing down time. Taking $\tau_s \propto T^{3/2}/n$ leads to

$$\frac{n_\alpha}{n} \propto B^{2/5}.$$

Similarly for the family members the ratio

$$\frac{V_\alpha}{V_{\mathrm{ALFVEN}}} \propto \frac{n^{1/2}}{B} \propto B^{-1/5}.$$

A third important o-particle parameter

$$\beta_\alpha = n_\alpha \varepsilon_\alpha / B^2 \propto B^0$$

and so is property invariant within the family.

Evidently the variations in o-particle physics are weak and perhaps negligible.

Qualifications

Discharges with constant β, G, and C and the same geometric variables require similarity in the sources of particles, momentum, and energy. The analysis covers all transport formulae derived on the basis of fully-ionized plasma physics, for example the Rebut-Lallia (1958) global scaling. All of these countless calculations of transport are by definition expressible in terms of β, G, and C.

Purely empirical scalings are not normally so expressible although minor changes in the indices, usually within the errors, will bring such scalings into line. For example in the cases where power laws are used and

$$B\tau_\varepsilon = B^w L^x n^y T^z$$

then it is readily shown that the law is expressible in terms of β, G, and C if

$$\frac{5w}{2} - 2x + 4y + z = 0.$$

Examination of the Goldston (1984) law shows for example that the addition of a weak density dependence to the basic result is all that is required, i.e.,

$$\tau_\varepsilon \propto I L^{1.38} P^{-1/2} n^{1/8}.$$

Bremsstrahlung, cyclotron and impurity radiation are excluded from the scaling. Consequently it is no surprise that the Murakami limit is also excluded.

Ignition Experiment Proposals

A wide range of ignition experiment proposals are on the table. Generally they have been conceived without regard to family membership as discussed earlier, but usually aim at achieving an ignited plasma at minimum cost based on one or several confinement scaling laws. Table 2 shows a sample set of such proposals. For each case the current has been calculated on the same basis, namely $q_c = 2$ and

$$I = 5a^2\varepsilon B/Rq_c(\text{MA, m, T}).$$

The plasma elongation ε is taken as 2 for all cases. As a confinement benchmark the Goldston scaling law is used, γ_g is the gain over Goldston confinement required in each case to ensure ignition. This is calculated assuming a pressure peaking factor of 3 and the depletion associated with $Z = 7$ light impurities corresponding to $Z_{\text{eff}} = 2$. The magnetic energy in the plasma volume is also tabulated. For comparable technologies and sophistication this can be regarded as a crude indicator of cost. Clearly machines such as ITER and PCSR with superconducting coils and blankets will be relatively much more expensive than copper-coiled devices designed to demonstrate ignition for a few confinement times.

	R(m)	a(m)	B(T)	I(MA)	A	γ_g	Magnetic Energy (GJ)
Ignitor	1.2	0.43	15	11.6	2.8	2.7	0.8
CIT	2.1	0.65	10	10	3.2	2.7	1.4
Ignitex Super	2.1	0.54	20	14	3.9	1.5	3.8
ET-n	5.1	1.27	7	11	4.0	2.0	6.3
ITER	5.8	2.0	5.1	18	2.9	2.0	10.0
JIT	*7.5*	*3.0*	*4.5*	*27*	2.5	1.7	21.5
PCSR	*9.6*	*2.4*	*6.0*	**18**	4.0	1.4	31.0

Table 2: Sample Ignition Experiment Proposals

It is noteworthy that the reactor, sized by technical constraints, requires only a modest improvement over Goldston. Close behind in this respect is the very high field large version of Ignitex where the main doubt remains its engineering feasibility.

Since these machines are not members of the same family we can tabulate their family characteristics, namely β, G, C, D, F, A, and b/a, noting that for all the proposals we have used the same value of q and isotopic mix (50:50). The results are shown in Table 3 where all values are normalized to those of an operating reactor. In the same table the normalized parameters for high temperature discharges in JET and TFTR and for high β experiments in DIIID are given. As expected these devices are all too small as reflected in the parameter G while the high β experiments are also very collisional (high C).

	β	G	C	D	F	A	b/a
Reactor at ignition	0.33	1.0	0.33	3.0	1.0	1.0	1.0
Ignitex Super at ignition	0.12	1.5	1.0	8.0	1.0	1.0	1.0
ET-n	0.5	3.2	0.6	6.2	1.0	1.0	1.0
CIT	1.2	5.4	2.0	4.4	1.0	0.67	1.0
ITER	1.4	2.1	1.2	1.5	1.0	0.72	1.0
JIT	1.5	1.2	1.1	0.8	1.0	0.62	1.0
JET (high temperature)	0.18	9.0	0.9	20	~ 0.1	0.67	0.8
TFTR (high temperature)	0.12	8.0	0.5	30	~ 0.1	0.78	0.5
DIIID (high beta)	2.0	90	360	2.3	?	0.62	1.0

Table 3: Family Characteristics of Proposed Ignition Experiments Normalized to an Operating Reactor

α-particle physics

Again since these proposed machines are not members of the same family the scaling of o-particle physics is different from that discussed earlier. The common feature of these machines is ignition so that

$$nT\tau_\epsilon = \text{constant}.$$

Using this relation we find the following scalings,

$$n_\alpha/n \propto T^{7/2},$$

$$\beta_\alpha \propto nT^{7/2}/B^2,$$

$$V_\alpha/V_{\mathrm{ALFVEN}} \;\propto\; n^{1/2}/B.$$

Table 4 shows these a-particle physics parameters for the proposed devices. It is evident that the extreme high field machine at the ignition point constitutes a very poor test of reactor-relevant o-particle physics.

	n_α/n $\left(\propto T^{7/2}\right)$	β_α $\left(\propto nT^{7/2}/B^2\right)$	$V_\alpha/V_{\mathrm{ALFVEN}}$ $\left(\propto n^{1/2}/B\right)$
Reactor at ignition	1.0	0.3	0.6
Ignitex Super	0.2	0.04	0.4
ET-n	1.0	0.4	0.6
ITER	1.0	1.4	1.2
JIT	1.0	1.5	1.2
CIT	1.0	1.2	1.1

Table 4: α-Particle Physics Parameters in Proposed Ignition Experiments Normalized to an Operating Reactor

Discussion

Note that if we insisted on the strict similarity discussed earlier then the auxiliary power required to reach ignition P_{AUX}, would scale as

$$P_{\mathrm{AUX}} \;\propto\; nTL^3/\;\tau_\varepsilon \;\propto\; B^{3/5},$$

while the power loading per unit area of wall P_ω, would scale like

$$P_\omega \;\propto\; P_{\mathrm{AUX}}/L^2 \;\propto\; B^{11/5}.$$

This somewhat negative view of high field systems is the result of insisting on strict similarity and the non-optimum isotope mix that is then required.

A more practical approach is to ignite the high field system at low temperature with the optimum isotope mix and then allow it to heat up and at the same time lean off the mixture so that eventually it is operating near a point of strict similarity with the reactor plasma.

Such a scenario demands that the high field system operates for many energy confinement times which may be difficult to achieve.

In the light of these arguments the proposed ignition experiments can be divided into four classes in order of increasing cost. The most inexpensive are the high field experiments such as Ignitor and Ignites which will demonstrate ignition but will not test the plasma physics appropriate to D-T reactors, next are the more moderate field devices such as CIT and ET-n which if they ignite can test much of the relevant physics for a few energy confinement times. JIT is a long pulse copper coil machine which can test much of the physics including exhaust and fuelling, but at an aspect ratio probably not feasible in a reactor. Finally ITER is designed to test both the physics and technology through the use of superconducting coils and a limited blanket. Again the aspect ratio is too small to be reactor relevant.

In terms of immediate action it is clear that the now extensive database on confinement should be cast in the form of $(B\tau_\varepsilon)$ as a function of β, G, C and the geometric variables q, ε, and A. With the data in this form it may be possible to show that similar discharges do indeed behave in the similar way described by Kadomtsev, it should also be possible to extract the functional dependence of (Br) on these parameters. If such a program is not successful it can only be because power deposition and fuelling profiles are not similar, or because radiation is important in the interior.

Finally we note that although the strict similarity arguments have been applied to the parameters of a realistic and perhaps undesirable steady state D-T system, the same procedure can be applied to any vision of a more desirable system that may evolve. Similarly one can scale forward from present experiments to members of the same family that would ignite. For example if JET reaches a thermonuclear F = 0.1 with $B = 3.4$ T, then it would ignite if the field was increased tenfold, the current by SO%, the density by 40, the temperature by 2.5 and the size reduced by a factor 6.3. Auxiliary heating power would need to be increased by 4. These numbers are totally impractical but they do describe an igniting relative of JET.

Acknowledgment

This work was supported by the Texas Atomic Energy Research Foundation.

References

1. Bickerton, R. J. and London, H., Proc. Phys. Soc. 72, 116 (1958).

2. Connor, J. and Taylor, J. B., Nuclear Fusion 17, 1047 (1977).

3. Goldston, R. J., Plasma Physics and Controlled Fusion 26, 87 (1984).

4. Kadomtsev, B. B., Journal Plasma Physics 1, 295 (1975).

5. Rebut, P. H., Lallia, P., and Watkins, M. L., Paper IAEA-CN-SO/DC1 Nice (1988).